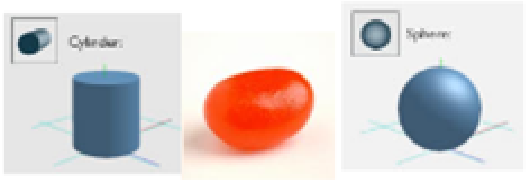
Investigation: Finding Volume of Composite Shapes

Dissecting shapes into more familiar shapes is one strategy used to determine the area or volume. For example, dissecting the circle into pieces and reassembling them allowed for the use of the formula for the area of a parallelogram. As you work on the following problems, look for answers to the following questions:

In what ways can simple shapes be used to calculate the volume of more complex shapes?

1. Marcelle and Haven wanted to enter the Jelly Bean Counting Contest. They both thought that finding the volume of the jelly bean would be helpful but were disagreeing on how to make the best estimate. Marcelle said that the jelly bean looked a lot like a cylinder and Haven thought it looked like a sphere.



   a. What do you think it looks like? Discuss with others.
   b. Marcelle measured the jelly bean and found that it was about 1.5 cm long and about 1 cm wide. Using his measurements, what will Marcelle estimate as the volume of the jelly bean?
   c. Haven cut the jelly bean in half and measured across from edge to edge several times. She found the mean of her measurements to be 1.3 cm. What will Haven estimate as the volume of the jelly bean?
   d. Compare the two estimates. Decide on an estimate for the volume of a jelly bean and justify your reasoning.

2. Marcelle and Haven thought that by finding the volume of a jelly bean they would be able to determine how many jelly beans were in the bucket (measuring 7cm in diameter and 10 cm in height); however, they are not sure how to proceed.
   a. Discuss in your group and develop a plan to share with Marcelle and Haven. Be specific as to what information is needed and how to use the information.
   b. What is the estimate for the number of jelly beans in the bucket?
   c. Marcelle thinks the estimate may be too high because the jelly beans do not fit tightly together. He estimates that 10% of the volume is filled with air and not with jelly beans. With this consideration, what is the new estimate for the number of jelly beans?
   d. How does the estimated number of jelly beans compare to Activity 5c in the previous investigation *Discovering Formulas for Area and Volume*? What would you estimate for the number of jelly beans? Explain your reasoning.

3. Modeling with a simple shape is a way to estimate volume. For each of the following items:
   a. Illustrate and describe what simple shape(s) could be used to model the item.
   b. Explain how the formulas for the simple shapes could be used to calculate the volume.

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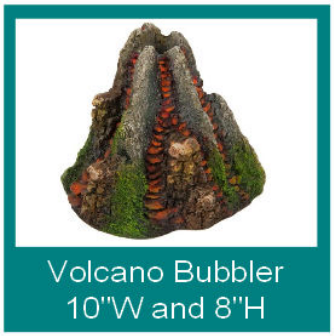
How many fish?

Cardinal Tetra

4. The number of fish that a fish tank can hold depends on the number of gallons of water. The typical rule is that there should be 1 gallon of water for every inch of fish length.
   a. The average length of a cardinal tetra is 2". Determine the number of cardinal tetras that can be supported in a fish tank that is 30"L x 12"W x 12"H. (1 gallon = 231 in^3)
   b. In setting up the tank, fresh water gravel should be layered on the bottom 2" deep and the water should be filled to one inch from the top. How does this change the maximum number of cardinal tetras recommended for the tank? Explain.
   c. Tank accessories can also change the amount of water. Consider the following items being placed into the tank and determine if the maximum number of cardinal tetras changes. Be prepared to share your reasoning with others.



Volcano Bubbler
10"W and 8"H

Two Tiki Totems
2.5"W and 5"H

Two Coral Orange Brains
4.5"L,1.6"W and 1.6"H

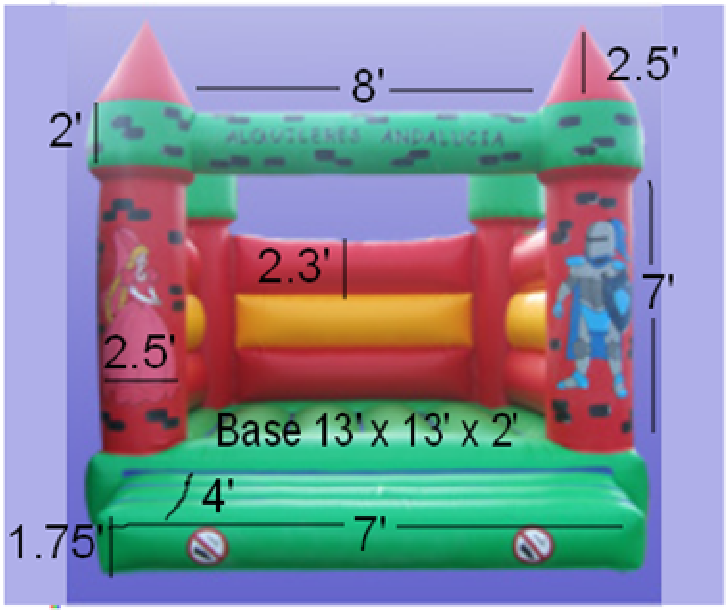
Summarize the Mathematics

In this investigation you solved problems by calculating the volume of complex shapes.

a) Describe a strategy for calculating the volume of complex shapes.
b) What simple shapes did you rely on to make your calculations?

Check Your Understanding

a) The Bounce Party Company will deliver, set up and remove inflatable rentals for special events. A 1.5HP blower is typically used to inflate the Turret Castle. This blower can produce 1290 cubic feet per minute. How long will it take to inflate the Turret Castle?



Investigation: Finding Volume of Composite Shapes (Teacher Notes)

This investigation should follow Investigation: Discovering Formulas for Area & Volume

NC CCSS Math 1: Quantities
- N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and origin in graphs and data displays.
- N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.

NC CCSS Math 1: Seeing Structure in Expressions
- A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*

NC CCSS Math 1: Geometric Measurement & Dimension
- G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
- G-GMD.2 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

1. Formulas for simple shapes have been provided in the Teacher Resources.
   a. Students should think about and discuss the reasoning of Marcelle and Haven.
   b. Marcelle thinks the jelly bean looks like a cylinder so he would use the formula $V = \pi r^2 h$ where the r=0.5 cm and h=1.5cm. The estimated volume of the jelly bean is approximately $\pi(0.5\text{cm})^2(1.5\text{cm})=1.2\text{cm}^3$.
   c. Haven thinks the jelly bean looks like a sphere so she would use the formula $V = \frac{4}{3}\pi r^3$ where r=0.65cm. The estimated volume of the jelly bean is approximately $\frac{4}{3}\pi(0.65cm)^3$ = 1.2cm³.
   d. Marcelle and Haven have estimated the same volume of the jelly bean to be 1.2cm³.

2.
   a. The volume of bucket divided by the volume of jelly bean will estimate the number of jelly beans in the bucket. The information needed is the radius of the bucket, height of the bucket, and the volume of the jelly bean.
   b. The volume of the bucket is $\pi(3.5\text{ cm})^2(10\text{ cm}) = 384.85\text{ cm}^3$. The estimated number of jelly beans in the bucket is approximately 384.85 cm³/ 1.2cm³ = 322 jelly beans.
   c. If 10% is air then 90% is jelly beans so 0.90(322) is approximately 290 jelly beans.
   d. How does the estimated number of jelly beans compare to Activity 5c in the previous investigation *Discovering Formulas for Area and Volume*? What would you estimate for the number of jelly beans? Explain your reasoning.
   e. The estimate is 30 jelly beans more. Students may decide on one of the two estimates or they may give other estimates such as average of the two estimates. The number of actual jelly beans in the bucket pictured in the Think About this Situation is unknown as the image was found on the internet.
3. An enlarged image of each shape can be found in the Teaching Resource. Students should illustrate and describe the shape. For part b, students should use formulas appropriate for their illustrated description in part a to determine how to calculate the estimated volume. Suggested simple shapes are listed below.
   i. trash can modeled by half a sphere and a cylinder
   ii. donut modeled by (outer cylinder – inner cylinder)
   iii. bottle modeled by a square pyramid and a sphere
   iv. grill modeled by a sphere
   v. stack of coins modeled by a cylinder

vi. toy modeled by half a sphere, three different size cylinders and ¾ of a sphere OR half a sphere and a cone.
vii. toolbox modeled by a rectangular prism and a hexagonal prism
viii. cone modeled by 3/4 of a cone

4.

a. The volume of the tank is (30 in)(12 in)(12 in) = 4320 in³. The number of gallons of water 4320 in³ / 231 in³ per gallon = 18.7 gallons. Considering 2" cardinal tetras, 18.7 gallons/2" fish per gallon = 9.35 or approximately 9 two inch fish.
b. The volume of the tank decreases to (30 in)(12 in)(9 in)=3240 in³ or 14 gallons. Thus, only 7 two in fish would be recommended.
c. The volcano bubbler is similar to a cone with radius 5" and height 8". The estimated volume is $\frac{1}{3}\pi(5in)^2 8in$ =209 in³.The Tiki Totems are similar to cylinders with radius of 1.25" and height 5". The estimated volume of one is $\pi(1.25in)^2 5in$ = 24.5 in³ and so two of the totems would be 2(24.5 in³) = 49 in³. The coral orange brain is similar to a rectangular prism or cylinder. If a rectangular prism is used with base dimensions of 4.5" by 1.6" and a height of 1.6" then the estimated volume is (4.5 in)(1.6 in)(1.6 in) = 11.52 in³. If a cylinder is used with a radius of 0.8" and height of 4.5" then the estimated volume is $\pi(10.8in)^2 4.5in$ = 9 in³. Two of the rectangular prisms would be approximately 2(11.52 in³) = 23 in³ and two cylinders would be 2(9 in³) = 18 in³. The volume of the tank with water rock and suggested water level has a volume of 3240 in³. The total volume for the accessories is 209 in³ + 49 in³ + (18 in³ or 23 in³) which means the volume in the tank decreases by either 276 in³ or 281 in³. Both values indicate about a gallon of water which would decrease the number of two inch fish by one thus recommending only 6 fish instead of 7. Another approach is to calculate the volume of the tank. It would be 2964 in³ or 2959 in³ which are both approximately 12.8 gallons and would hold 6 two inch fish.

Summarize the Mathematics

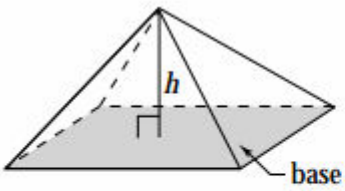
a) Dissecting the shapes into simple shapes allows for the use of the corresponding volume formulas to calculate the volume of more complex shapes.
b) cylinders, prisms, pyramids, spheres and cones

Check Your Understanding

a) Calculate each simple shape and then add them together.
   i. The base step is (1.75 ft)(7 ft)(4 ft) = 49 ft³
   ii. The base is (13 ft)(13 ft)(2 ft) = 338 ft³
   iii. The three walls are 3(6.9 ft)(2.5 ft)(2.3 ft) = 129.4 ft³
   iv. The four turrets are 4π(1.25 ft)²(7 ft) = 137.4 ft³
   v. The tops of the turrets are 4π(1.25 ft)²(2 ft) = 39.3 ft³
   vi. The top piece connecting the front turrets is π(1 ft)²(8 ft)=25.1 ft³
   vii. The cones on top of the turrets are $4(\frac{4}{3}\pi(1ft)^2(2.5ft)$ = 10.5 ft³

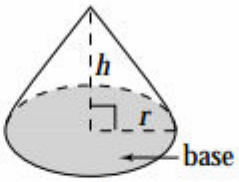
The total volume of the Turret Castle is 728.7 ft³. At a rate of 1290 ft³ per minute it would take approximately 1290 ft³ / 728.7 ft³ per minute = 1.8 minutes or 1 minute and 48 seconds.

Formulas of Simple Shapes



$V = \frac{1}{3}Bh$

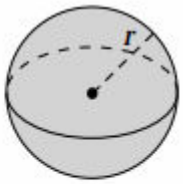
Pyramid



$V = \frac{1}{3}Bh$

$V = \frac{1}{3}\pi r^2 h$

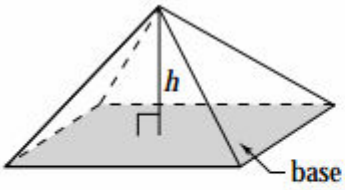
Cone



$V = \frac{4}{3}\pi r^3$

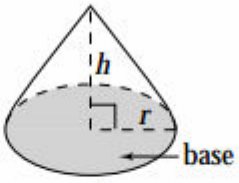
Sphere

Formulas of Simple Shapes



$V = \frac{1}{3}Bh$

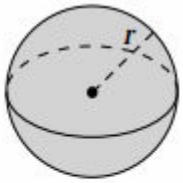
Pyramid



$V = \frac{1}{3}Bh$

$V = \frac{1}{3}\pi r^2 h$

Cone



$V = \frac{4}{3}\pi r^3$

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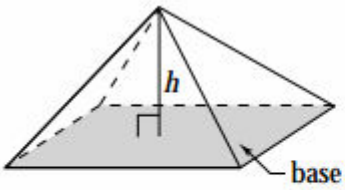
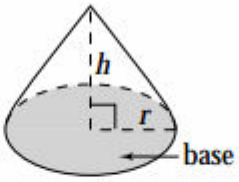
Formulas of Simple Shapes



$V = \frac{1}{3}Bh$

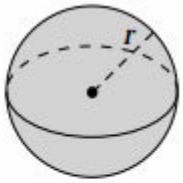
Pyramid



$V = \frac{1}{3}Bh$

$V = \frac{1}{3}\pi r^2 h$

Cone



$V = \frac{4}{3}\pi r^3$

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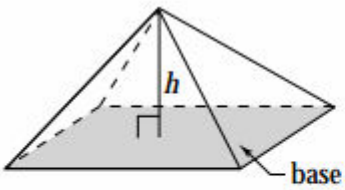
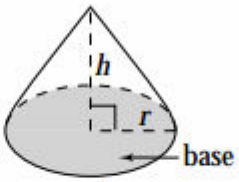
Formulas of Simple Shapes



$V = \frac{1}{3}Bh$

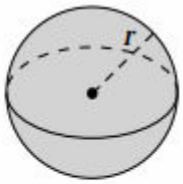
Pyramid



$V = \frac{1}{3}Bh$

$V = \frac{1}{3}\pi r^2 h$

Cone



$V = \frac{4}{3}\pi r^3$

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Investigation: Finding Volume of Composite Shapes
Problem #3

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